MAHARASHTRA AGRICULTURAL UNIVERSITIES EXAMINATION BOARD, PUNE
SEMESTER END EXAMINATION

B.Sc.(Hons.) Agriculture/B.Sc.(Hons.) (Forestry)/ B.Sc. (Hons.) A.B.M.

Semester	: I (New)	Term	: I	Academic Year	: 2017-18
Course No.	: MATH 111	Title	: Elementary Mathematics		
Credits	: 2(1+1)				
Day & Date	: Friday, 29.12.2017	Time	: 09.00 to 11.00	Total Marks	: 40

Note : 1. Solve ANY EIGHT questions from SECTION "A".
2. All questions from SECTION "B" are compulsory.
3. All questions carry equal marks.
4. Draw neat diagrams wherever necessary.

SECTION "A"

Q.1 If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A$ is a scalar Matrix.

Q.2 Evaluate

$$\begin{vmatrix} 2 & 6 & 2 \\ 5 & 9 & 4 \\ 8 & 12 & 7 \end{vmatrix}$$

Q.3 Find the acute angle between the lines $2x + y - 1 = 0$ and $3x + y + 4 = 0$

Q.4 Find the area of a field by Simpson's rule, if the ordinates are 2, 7, 9, 15, 21, 30, 12 meters and common distance is 33 meters.

Q.5 Find the equation of straight line passing through the two points P (1, 2) and Q (4, 3).

Q.6 Explain any four types of function with one example each.

Q.7 Explain the ordinates of the curve and common distance between the ordinates. State the Simpson's 1/3 rd rule.

Q.8 Find derivatives of the following w.r.t.x. (Any Two)

1) $x^2 \tan x$ 2) $\cos^3 x$ 3) $(\sin x) / (e^x)$

Q.9 Evaluate the following definite integrals. (Any Two)

1) $\int_2^5 \left(\frac{1}{x}\right) dx$ 2) $\int_1^3 x^2 \, dx$ 3) $\int_0^1 2^x \, dx$

Q.10 Evaluate the limit $- \lim_{x \to 3} \dfrac{x^2 - 7x + 12}{x^2 - 5x + 6}$

(P.T.O.)

SECTION "B"

Q.11 State True or False.

1) Limit of product of two functions is equal to product of their limits.

2) The point (3, –4) lies in the third quadrant.

3) Simpson's rule is applied only if the numbers of ordinates are odd.

4) A square matrix 'A' is said to be non singular if $|A| = 0$

Q.12 Select the correct answer.

1) The area bounded by the curve $y = f(x)$, X axis and the lines $x = a$ and $x = b$ is_____.

a) $\int_a^b y dx$ b) $\int y dx$ c) $\int_b^a y dx$ d) $\int_a^b dx$

2) A square matrix in which all the non diagonal elements are zero is called_____.

a) Scalar matrix b) Diagonal matrix c) Identity matrix d) Null matrix

3) The radius of the circle $x^2 + y^2 = 49$ is _____.

a) 49 b) 4 c) 9 d) 7

4) $\int a^x dx =$_____ + c.

a) $a^x \log a$ b) a^x c) $a^x/\log a$ d) $\log a$

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MAHARASHTRA AGRICULTURAL UNIVERSITIES EXAMINATION BOARD, PUNE
SEMESTER END EXAMINATION

B.Sc. (Hons.) Agriculture

Semester	: I (New)	Term: I	Academic Year: 2017-2018
Course No.	: Math-111	Title	: Elementary Mathematics
Credits	: 2 (2+0)		
Day &Date	:	Time : 3 hrs	Total Marks : 80

Note: 1. Solve ANY EIGHT questions from Section "A".
2. All questions from Section "B" are compulsory.
3. All questions carry equal marks.
4. Draw neat diagrams wherever necessary.
5. Correct answer obtained by any logically correct mathematical method weights full marks.

MODEL ANSWER

SECTION-A

Q. 1 b) If $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}$, show that A^2-4A is a scalar Matrix.

Ans: $A^2 = A \times A = \begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}$ (1 Mark)

$= \begin{pmatrix}9 & 8 & 8\\ 8 & 9 & 8\\ 8 & 8 & 9\end{pmatrix}$ (1 Mark)

$A^2-4A = \begin{pmatrix}9 & 8 & 8\\ 8 & 9 & 8\\ 8 & 8 & 9\end{pmatrix} - 4\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}$ (1 Mark)

$A^2-4A = \begin{pmatrix}5 & 0 & 5\\ 0 & 5 & 0\\ 0 & 0 & 5\end{pmatrix}$ (1 Mark)

Q.2 a) Evaluate

$$\begin{vmatrix}2 & 6 & 2\\ 5 & 9 & 4\\ 8 & 12 & 7\end{vmatrix}$$

Ans : $= 2(63-48) - 6(35-32) + 2(60-72)$ (2 Mark)

$= 2(15) - 6(3) + 2(-12)$ (1 Mark)

$= 30 - 18 - 24$

$= 30 - 42$

$= -12$ (1 Mark)

Q.3 Find the acute angel between the lines $2x + y - 1 = 0$ and $3x + y + 4 = 0$.

Ans : Given equation of lines are

$2x + y - 1 = 0$ .... (i)

$3x + y + 4 = 0$ .... (ii)

Slope of line (i) is $m_1 = \frac{-2}{1} = -2$ and $m_2 = \frac{-3}{1} = -3$ (1 Mark)

Let θ be the acute angle between the lines (i) and (ii), then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad (1 \text{ Mark})$$

$$\tan\theta = \left|\frac{-2-(-3)}{1+(-2)(-3)}\right| \quad (1 \text{ Mark})$$

$$\tan\theta = \left|\frac{-2+3}{1+6}\right|$$

$$\tan\theta = \left|\frac{1}{7}\right|$$

$$\theta = \tan^{-1}\left(\frac{1}{7}\right) \quad (1 \text{ Mark})$$

Q.4. Find the area of a field by Simpsons rule , if the ordinates are 2, 7, 9, 15, 21, 30, 12 meters and common distance 33 meter.

Ans : Here $P_1 = 2$, $P2 = 7$, $P_3 = 9$, $P_4 = 15$, $P_5 = 21$, $P_6 = 30$, $P_7 = 12$ and C.D. d = 33 meters

Area = $(d/3)\,[(P_1 + P_7) + 4\,(P_2 + P_4 + P_6) + 2\,(P_3 + P_5)]$ (1 Mark)

Area = $(33/3)\,[(2 + 12) + 4\,(7 + 15 + 30) + 2\,(9 + 21)]$ (1 Mark)

Area = $(11)\,[14 + 4\,(52) + 2\,(30)]$

Area = $(11)\,(14 + 208 + 60)$ (1 Mark)

Area = $(11)\,(280)$

Area = 3102 sq.meters (1 Mark)

Q.5. b) Find the equation of straight line passing through the two points P (1, 2) and Q (4, 3).

Ans : By two point form Equation of line is ,

$$y - y_1 = (y_2 - y_1)\,\frac{x - x_1}{x_2 - x_1} \quad (1 \text{ Mark})$$

$$y - 2 = (3 - 2)\,\frac{x - 1}{4 - 1} \quad (1 \text{ Mark})$$

$$y - 2 = \frac{x - 1}{3} \quad (1 \text{ Mark})$$

$$3y - 6 = x - 1$$

$$x - 3y + 5 = 0 \quad (1 \text{ Mark})$$

Q.6. a) Explain any four types of function with one example each. (1 Mark for each function and maximum 4 marks)

Ans : 1. Algebraic function or Polynomial function

A function f of the form $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n{}_n$ is called algebraic function. Where $a_0, a_1, a_2, \ldots\ldots$ an are real numbers and n is an integer.

Ex. $f(x) = 3x^2 + 2x + 3$ is a algebraic function.

2. Exponential function

A function of the form $f(x) = a^x$ is called exponential function where a is a real positive number i.e. $a > 0$.

Ex. i) $y = f(x) = 2^x$

ii) $y = f(x) = e^x$ where $e = 2.7183$

3. Logarithmic function

A function defined by, $y = f(x) = \log_a x$, is called logarithmic function to the base a, where a is any real positive number other than 1.

Ex. $y = f(x) = \log_5 625$

4. Trigonometric function

A function defined by, $f(x) = \sin x$, $f(x) = \cos x$, $f(x) = \tan x$, $f(x) = \cot x$, $f(x) = \sec x$, $f(x) = \operatorname{cosec} x$, are called trigonometric function

5. Inverse function

If $y = f(x)$ be a function the then function of the form $f^{-1}(y) = x$ is called a inverse function of $y = f(x)$

Ex. Let $f(x) = y = 2x + 1$

$x = (y-1)/2$

Then $f^{-1}(y) = x = (y-1)/2$

Thus $f^{-1}(y)$ $x = (y-1)/2$ is a Inverse function of $y = f(x) = 2x + 1$

6.Inverse Trigonometric function

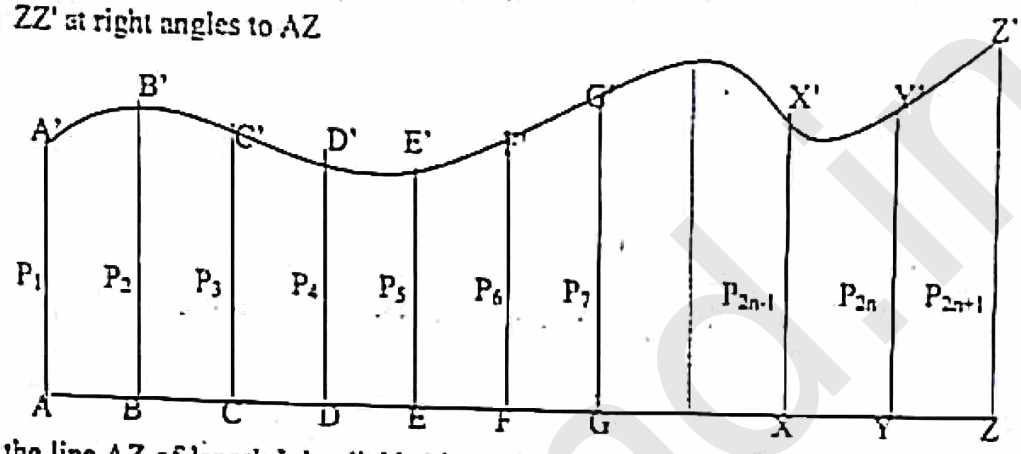
If $y = \sin x$ be a function, then the function of the form $x = \sin^{-1} y$ is called a Inverse trigonometric function where, x, the angles are expressed in radians and y, is a real number.

Ex. $y = \sin x$, Let $x = \pi/2$ $\therefore y = \sin \pi/2 = 1$

$x = \sin^{-1} 1 = \pi/2$

Q.7. a) Explain the ordinates of the curve and common distance between the ordinates also state the Simpsons 1/3 rd rule.

Ans : Consider a figure bounded by a curve A' G' Z' a straight line AZ and two straight lines AA' and ZZ' at right angles to AZ



Let the line AZ of length L be divided into even number (2n) of parts of equal length, AB = BC = CD = ... = XY = YZ. The length of these parts is called the common distance (d).

$\therefore$ Common distance = $d = \frac{L}{2n}$ where n = 1, 2, 3, ... n (1 Mark)

At the point of division B, C, D ... G, ... X, Y, Z draw BB', CC', DD', ... XX', YY' and ZZ' perpendiculars to AZ to meet the curve in B', C', D', ... X', Y'. These perpendiculars are called the ORDINATES to curve A' G' Z'. They are denoted by $P_1, P_2, P_3, \ldots P_{2n-1}, P_{2n}, P_{2n+1}$.
(1 Mark)

Simpson's Rule :

This rule is used to find approximate area (A) of a figure bounded by a curve, when the lengths of an odd number of ORDINATES and their common distance is given.

Area = $\frac{1}{3}$ Common distance [(first ordinate + last ordinate)

+ 2 (sum of remaining odd ordinates) + 4 (sum of even ordinates)]

$$A = \frac{d}{3}\left[(P_1 + P_{2n+1}) + 2(P_3 + P_5 + P_7 + \ldots + P_{2n-1}) + 4(P_2 + P_4 + P_6 + \ldots + P_{2n})\right]$$ (2 Marks)

Q.8. find the derivatives of the following w. r. t. x (Any Two) (1 Mark for each step)

1) $x^2 \tan x$ 2) $\cos^3 x$ 3) $\sin x / e^x$

Ans : 1) $x^2 [d/dx(\tan x)] + \tan x [d/dx(x^2)] = x^2 \sec^2 x + \tan x (2x)$

2) $3\cos^2 x [d/dx(\cos x)] = -3\cos^2 x \sin x$

3) $[e^x d/dx(\sin x) - \sin x \, d/dx(e^x)] / (e^x)^2 = (e^x \cos x - \cos x \, e^x) / (e^{2x})$

Q.9 Evaluate the following definite integrals. (Any two)

1) $\int_{2}^{5} (1/x)\,dx$ 2) $\int_{1}^{3} x^2\,dx$ 3) $\int_{0}^{1} 2^x\,dx$

Ans :

1) $\int_{2}^{5} (1/x)\,dx = [\log x]_{2}^{5}$ (1 Mark)

$= \log 5 - \log 2 = \log (5/2)$ (1 Mark)

2) $\int_{1}^{3} x^2\,dx = [x^3/3]_{1}^{3}$ (1 Mark)

$= (1/3)(3^3 - 1^3) = (9-1)/3 = 8/3$ (1 Mark)

3) $\int_{0}^{1} 2^x\,dx = [2^x / \log 2]_{0}^{1}$ (1 Mark)

$= (2^1 / \log 2) - (2^0 / \log 2) = (2-1)/\log 2 = 1/\log 2$ (1 Mark)

Q.10 Evaluate the Limit $\lim_{x \to 2} \frac{x^2 - 7x + 12}{x^2 - 5x + 6}$

Ans : $L = \lim_{x \to 2} \frac{x^2 - 7x + 12}{x^2 - 5x + 6}$

$= \lim_{x \to 3} \frac{(x-3)(x-4)}{(x-3)(x-2)}$ (1 Mark)

$= \lim_{x \to 3} \frac{x-4}{x-2}$ (1 Mark)

$= \frac{3-4}{3-2}$ (1 Mark)

$= -1$ (1 Mark)

SECTION-B

Q.11 1) True

2) False

3) True

4) False

Q.12. Select the correct answer

1] The area bounded by the curve $y = f(x)$, X axis and the lines $x = a$ and $x = b$ is $\int_a^b y\,dx$

2. A square matrix in which all the non diagonal elements are zero is called Diagonal matrix

3. The radius of the circle $x^2 + y^2 = 49$ is 7

4. $\int a^x\,dx = a^x / \log a + c$