MAHARASHTRA AGRICULTURAL UNIVERSITIES EXAMINATION BOARD, PUNE
SEMESTER END EXAMINATION
B.Sc. (Agri./Hort./For.)

Semester : 1 st (New) Academic Year : 2015-16
Course No. : MATH-111 Course Title: Mathematics (Deficiency Course)
Credits : 2(1+1) Total Marks : 80
Day & Date : Time:

MODEL ANSWER

Note: 1. Solve ANY EIGHT questions from SECTION "A"
2. All questions from Section "B" are compulsory.
3. All questions carry equal marks.
4. Draw neat diagrams wherever necessary.

SECTION "A"

Q1. a). Ans: The Simpson's Rule states that "Add together the first and last ordinates; twice the sum of even ordinates four times the sum of remaining odd ordinates and multiply the result by 1/3 of the common distance. (With Diagram)]

b). Ans: Given ordinates p1 = 2, p2 = 7, p3 = 9, p4 = 15, p5 = 21, p6 =30, p7 = 12 and Common distance d = 33 mtrs.

Total Area of a Curvilinear Figure is A

Therefore, $A = d/3\{(p1+p7) + 2(p2+p4+p6)+4(p3+p5)\}$

$A = 33/3\{(2+12) + 2(7+15+30) + 4(9+21)\}$

Therefore total area = A = 2618 Sq. mtrs.

Q.2. a). Ans: Let M/N be the fraction, and suppose

$x = \log M$ and $y = \log N$;

so that $a^x = M$ and $a^y = N$

$\therefore \quad M/N = a^x/a^y = a^{x-y}$

By definition, $\log M/N = x - y$

Hence proved $\log M/N = \log M - \log N$

b). Ans: Given N = 125 $a = 5\sqrt{5}$

By definition $a^x = N$

$(5\sqrt{5})^x = 125$

$(5\sqrt{5})^x = 5^3$

$(5^1.5^{1/2})^x = 5^3$

$(5^{1+1/2})^x = 5^3$. Therefore $x = 2$

Q.3. a). Ans: Given Quadratic equation is $ax^2 + bx + c = 0$

and the two roots are α and β

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\therefore \quad \alpha . \beta = \left\{\frac{-b+\sqrt{b^2-4ac}}{2a}\right\}.\left\{\frac{-b-\sqrt{b^2-4ac}}{2a}\right\}$$

Hence $\alpha . \beta = c / a$

b). Ans: Given quadratic equation is $x^2 - 4x + 3 = 0$

$\therefore (x - 3)(x - 1) = 0$

$(x - 3) = 0$ or $(x - 1) = 0$

$x = 3$ or $x = 1$

Therefore the solution of quadratic equation is {3 , 1}

Q.4. a). Ans: The theorems will be in the form of mathematical notations as f(x) and g(x) are two functions.

1. The limit of sum of two functions is equal to sum of their limits.
2. The limit of difference of two functions is equal to difference of their limits.
3. The limit of product of two functions is equal to product of their limits.
4. The limit of fraction of two functions is equal to fraction of their limits.

b). Ans: Given $\lim_{x\to 2} \frac{x^7-128}{x^5-32}$

$$= \lim_{x\to 2} \frac{x^7-128/x-2}{x^5-32/x-2} = \lim_{x\to 2} \frac{x^7-2^7/x-2}{x^5-2^5/x-2}$$

$$= \frac{7(2)^6}{5(2)^4} = \frac{28}{5}$$

Q.5. a). Ans:

1. $d/dx\,(2x^3 - 3x^2)$,

$= 2\, d/dx(x^3) - 3\, d/dx(x^2)$

$= 2.3x^2 - 3.2x$

$= 6x^2 - 6x$

2. $y = x^2 . \sin x$

$dy/dx = x^2 . d/dx(\sin x) + \sin x . d/dx(x^2)$

$dy/dx = x^2 . \cos x + \sin x . 2x$

b). Ans: Let δx be a small increment in x and δy be the corresponding increments in u and y respectively.

Then $\delta x \to 0$, $\delta u \to 0$ we have $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} . \frac{\delta u}{\delta x}$

By taking the limit as $\delta x \to 0$ from both sides we have

$\lim_{\delta u\to 0} \frac{\delta y}{\delta u}$ $\quad \lim_{\delta x\to 0} \frac{\delta u}{\delta x}$ is exists ,finite and equals $\frac{dy}{du}$ and $\frac{du}{dx}$

As u is a differentiable of function of x & y is a differentiable function of

Therefore the R.H.S. exists and finite. Hence the L.H.S. also exists finitely.

Hence proved $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Q.6. a). Ans: Properties of Determinant (with one example)

1. By interchanging the rows into columns and columns into rows the value of the determinant doesn't change.
2. If any two rows or columns of the determinant are interchanged then the value of the determinant is changed by its sign.
3. If any two rows or columns of the determinant are identical then value of the determinant is vanishes or zero.
4. If anyone row or column of the determinant is multiplied by the same factor then it should be multiply to the whole determinant.

b). Ans: Given $A = \begin{vmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 5 & -3 & 4 \end{vmatrix}$

$A = 2\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} - 3\begin{vmatrix} 0 & 2 \\ 5 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 5 & -3 \end{vmatrix}$

$A = 2\{(1x4) - (-3x2)\} - 3\{(0x4)-(5x2)\} + 1\{(0 x-3) - (5x1)\}$

$A = 45$

Q.7. a).Ans: Given Centre (h, k) = (3, 0) and radius r = 7

Equation of Circle is $(x - h)^2 + (y - k)^2 = r^2$

$\therefore \quad (x - 3)^2 + (y - 0)^2 = 7^2$

$x^2 + y^2 - 6x - 40 = 0$

This is the required equation of circle.

b). Ans: Given equation of circle is $x^2 - y^2 + 4x - 6y = 0$

Centre of the circle is (-g,-f) Radius of the circle is $\sqrt{g^2 + f^2 - c}$

$2g = 4$ and $2f = -6$ $\qquad r = \sqrt{2^2(-3^2) - 0}$

$g = 2$ and $f = -3$ $\qquad r = \sqrt{13}$

Therefore centre of the circle is (-2,3) and radius of the circle is $\sqrt{13}$

Q.8. a). Ans: Given points (x_1, y_1) =(1, 2) and (x_2, y_2) = (4, 3)

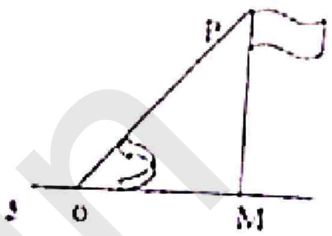
Therefore equation of straight line passing through two points is

$(y - y1) = \frac{y2-y1}{x2-x1}(x - x1)$

$(y - 2) = \frac{3-2}{4-1}(x - 1)$ & $\therefore$ $x - 3y + 5 = 0$ is required equation of straight

b).Ans: Let MP represent flagstaff and 'o' be the point from which the angle of Elevation is taken.

Then OM = 75 meters and $\angle MOP = 30^0$



Since PMO is a right angle, we have

$\frac{MP}{OM} = \tan MOP = \tan 30^0 = \frac{1}{\sqrt{3}}$

$MP = \frac{OM}{\sqrt{3}} = \frac{75}{\sqrt{3}} = \frac{75\sqrt{3}}{3} = 25\sqrt{3} = 25 \times 1.73 = 43.25$ meters

Q.9. a). Ans: Put $t = f(x)$ therefore $\frac{dt}{dx} = f'(x)$, $dx = \frac{dt}{f'(x)}$

$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{f'(x)}{t} \cdot \frac{dt}{f'(x)} = \int \frac{dt}{t} = \log(t) + c = \log[(f(x)] + c$

b). Ans: $\int Sin(2x + 7)dx$. 2. $\int (4x^3 + 3x^2 + 2x + 1)dx$

Put $t = 2x + 7$, $\therefore \frac{dx}{dt} = 2$. $\int (4x^3\ dx + \int (3x^2)dx + \int (2x)dx + \int 1dx$

$dx = \frac{dt}{2}$ $= 4\frac{x^4}{4} + 3\frac{x^3}{3} + 2\frac{x^2}{2} + x$

$\therefore \int \frac{\sin(t)}{2}dt$ $= x^4 + x^3 + x^2 + x$

$= \{-Cos(t)/2\}$

$= - Cos(2x+7)/2$

Q.10. a).Ans: Let $\int f(x)dx = F(x)$

$\therefore \int_a^b f(x)dx = F(x)]_a^b = F(b) - F(a)$..........(1)

R.H.S. $= -\int_b^a f(x)dx = [-F(x)]_b^a = -\{F(a) - F(b)\} = F(b) - F(a)$........(2)

From (1) and (2), $\int_a^b f(x)dx = -\int_b^a f(x)dx$

b). Ans: 1. $\int_1^2 \left(\frac{1}{x}\right)dx$ 2. $\int_0^1 Sinx\ dx$

$= \log 2 - \log 1$ $= \{-Cosx\}_0^1$

$= \log 2/1 = \log 2$ $= \{-Cos1 + Cos0\} = 1$

SECTION "B"

Q.11. Correct the following sentence if necessary and rewrite it.

1. The radius of the circle $x^2 + y^2 = 25$ is 25. (False, Radius r = 5)
2. The derivative of a constant is 1. (False, zero)
3. The quadratic equation has more than two roots. (False, cannot)
4. Integration and derivative are similar processes. (False, Inverse)
5. Tan $45^0 = 1$. (True)
6. e^x is a logarithmic function.(False, e^x is a exponential function)
7. The limit of a function is unique. (True)
8. Simpson's rule can be applied only when even ordinates are given. (False, Odd)

Q.12. Fill in the blanks.

1. $5x^2 - 1 = 0$ is ------------equation. (quadratic equation)
2. The point (0,-4) is lies in --------------quadrant. (Second)
3. The derivative of logx is ------------------ (1/x)
4. The logarithm of base itself is ----------- (one)
5. The equation of y axis is ---------------- (x = 0)
6. d/dx (Cos x) = ------------ (- Sin x)
7. The condition of two lines to be parallel is ---------------- (m1= m2)
8. If the height and distance is equal then the angle of elevation is --------- (acute)

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